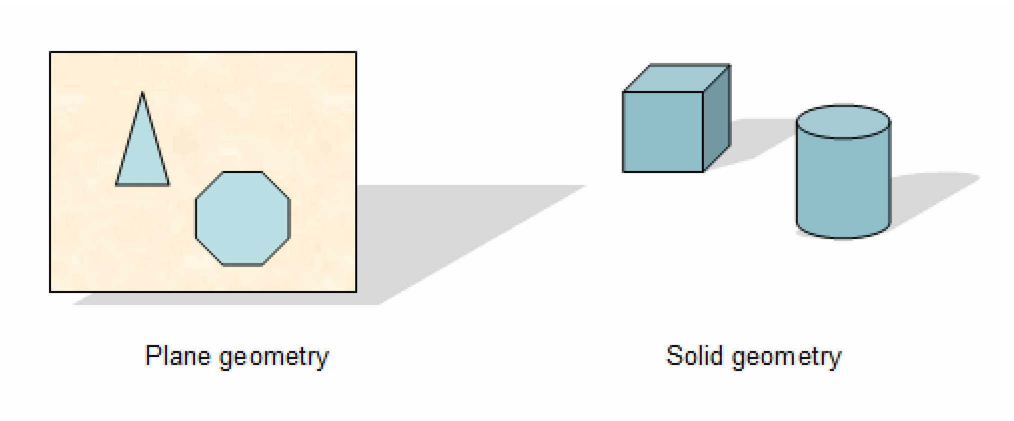
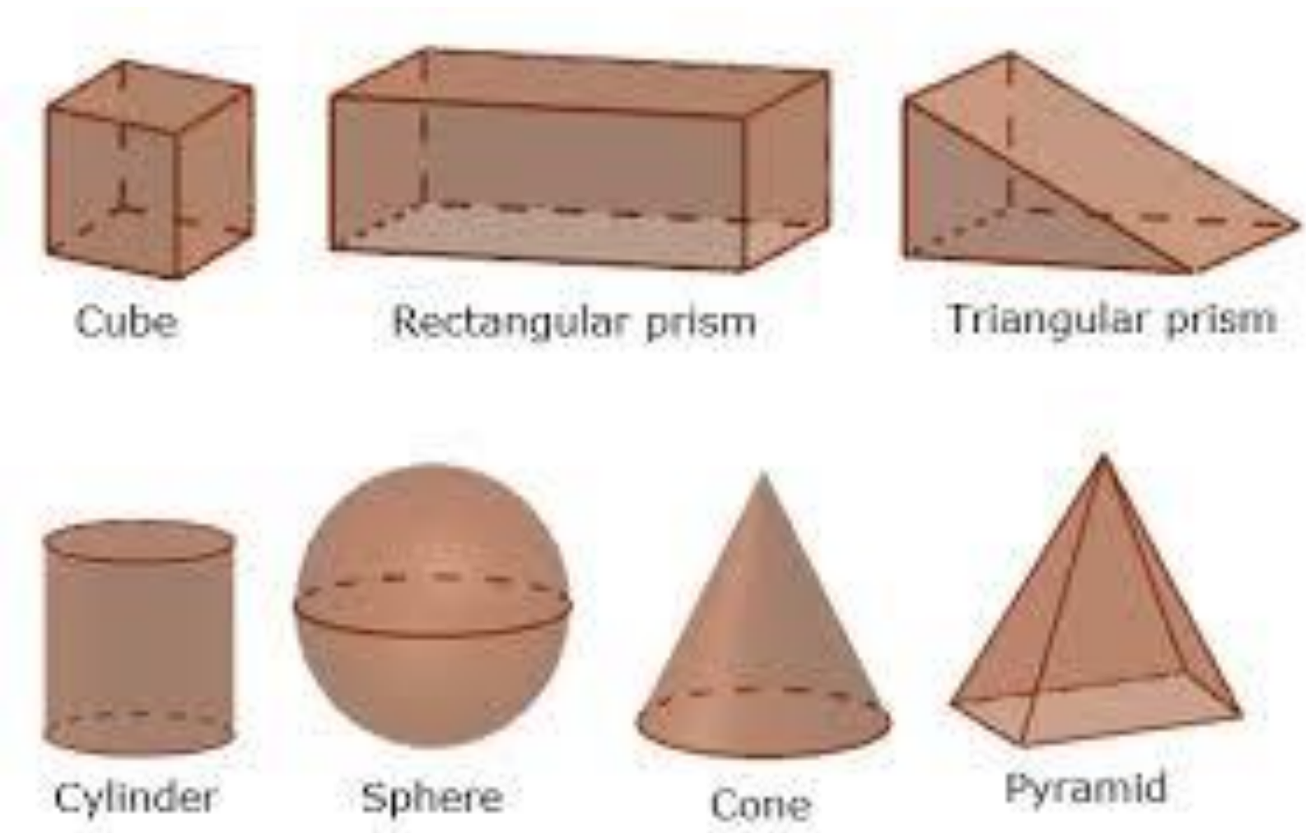
INTRODUCTION

The study of geometry can be broken into two broad types, plane geometry, which deals with only two dimensions, and solid geometry which allows all three dimensions.

Surface: In geometry, the boundary of any three dimensional solid.




Cube
Rectangular prism
Triangular prism
Cylinder
Sphere
Cone
Pyramid

Plane: A plane is a surface such that if any two points are taken on it, the line joining them lies wholly on the surface.

Note: Every equation of the first degree in x,y,z represents a plane.

The equation to every plane is of the first degree in x,y,z.

Normal form of the plane

Let the equation to the plane be $ax + by + cz + d = 0, a^2 + b^2 + c^2 \neq 0$

We can take $d \geq 0 \; or \; d \leq 0$.

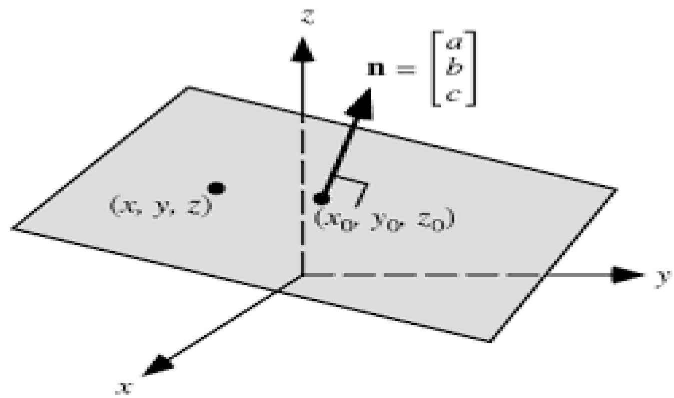
The normal form of the equation to the given plane is

$$\pm \frac{ax}{\sqrt{a^2+b^2+c^2}} \pm \frac{by}{\sqrt{a^2+b^2+c^2}} \pm \frac{cz}{\sqrt{a^2+b^2+c^2}} = \mp \frac{d}{\sqrt{a^2+b^2+c^2}} \; (d \geq 0 \; or \; d \leq 0)$$

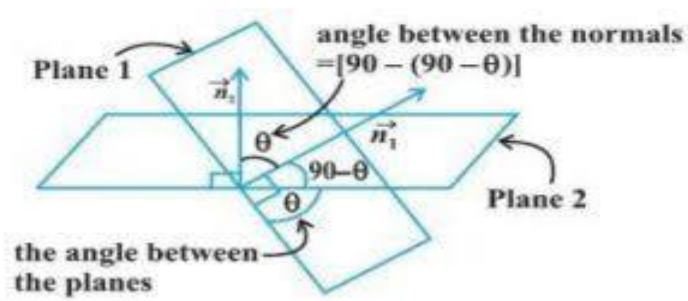
Distance of the origin from the plane $ax + by + cz + d = 0$ is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}$.

First degree equation in x,y,z without constant term $\Leftrightarrow$ plane is passing through the origin.

Consider the equation $\mathrm{l}x + my + nz = \mathrm{p} \; (\mathrm{l} \neq 0, m \neq 0, n \neq 0)$ of a plane direction cosines of a normal to it are l,m,n.



plane



Angle between two planes

Angles between two planes

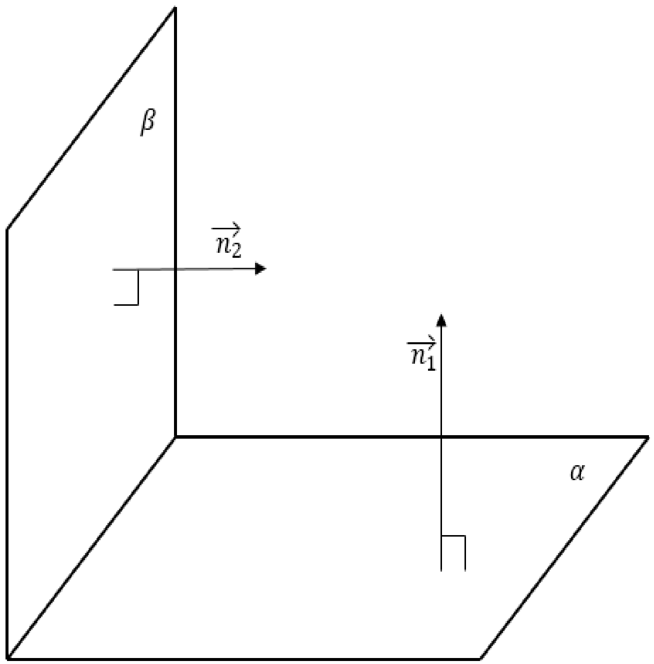
Angles between two planes $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$ are equal to the angles between their normals.

$$\theta = \cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right)$$, the other angle between the planes is $180^{\circ} - \theta$

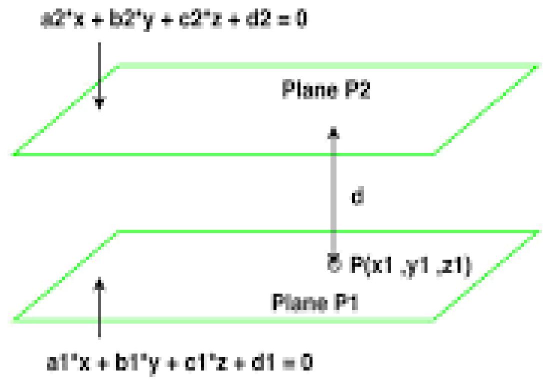
- The planes are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2=0$
- The two planes are parallel if $a_1{:}a_2=b_1{:}b_2=c_1{:}c_2$
- The equation of the plane parallel to $ax + by + cz + d = 0$ is $ax + by + cz + k = 0$

The equation of the plane through the point (x_1, y_1, z_1) and parallel to the plane $ax + by + cz + d = 0$ is $ax + by + cz + d = a_1x + b_1y + c_1z$.

- Equation of the plane making intercepts a,b,c on the coordinate axes is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c}=1$



Perpendicular planes



Parallel planes

The equation of the plane through the point (x_1, y_1, z_1) and perpendicular to the line with d.rs.(a,b,c) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$

Perpendicular Distance of a point from the Plane

The distance of (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ i.e., the length of the perpendicular from the point to the plane is $\frac{|ax_1+by_1+cz_1|}{\sqrt{a^2+b^2+c^2}}$.

Distance between parallel planes

Distance between parallel planes $ax + by + cz + d_1 = 0$, $ax + by + cz + d_2 = 0$ is

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

Problems

1. Find the equation of the plane through (4,4,0) and perpendicular to the planes x+2y+2z=5 and 3x+3y+2z-8=0

Sol: The equation of the plane passing through the given point (4,4,0) with d.rs. a,b,c is

$a(x-4)+b(y-4)+c(z-0)=0$

it is perpendicular to the plane x+2y+2z=5 and 3x+3y+2z-8=0

a+2b+2c=0,3a+3b+2c=0 by solving these two equations we get a=-2,b=4,c=-3

The equation of the plane is -2(x-4)+4(y-4)-3z=0

2x-4y+3z+8=0

2. Find the angles between the planes 2x-3y-6z=6,6x+3y-2z=18

Sol: let θ be one of the angles between the given planes

$$\therefore \theta = \cos^{-1}\left(\frac{2(6)-3*3-6*(-2)}{\sqrt{4+9+36\sqrt{36+9+4}}}\right)$$

$$\theta = \cos^{-1}\left(\frac{14}{59}\right)$$

The other angle between the planes is 180-$\cos^{-1}\left(\frac{14}{59}\right)$

3. Show that the equation of the plane through the points (2,2,-1),(3,4,2),(7,0,6) is 5x+2y-3z-17=0

Sol: The equation of the plane passing through the point(2,2,-1) with d.rs.a,b,c is

$$a(x-2)+b(y-2)+c(z+1)=0$$

since it passes through the points (3,4,2), (7,0,6)

a+2b+3c=0,5a-2b+6c=0

by solving these two equations we will get the equation of the plane is 5x+2y-3z-17=0.

4. A plane meets the coordinate axes A,B,C. If the centroid of $\Delta ABC\ is\ (a,b,c)$. Show that the equation to the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}$=3.

Sol: The equation of the planes which meets the coordinate axes at A=$(x_1,0,0), B=(y_1,0,0), C=(z_1,0,0)$ is $\frac{x}{x_1}+\frac{y}{y_1}+\frac{z}{z_1}$=1-------(1)

The centroid of $\Delta ABC\ (a,b,c)=\left(\frac{x_1}{3},\frac{y_1}{3},\frac{z_1}{3}\right)$.

Then x_1=3a, y_1=3b, z_1=3c

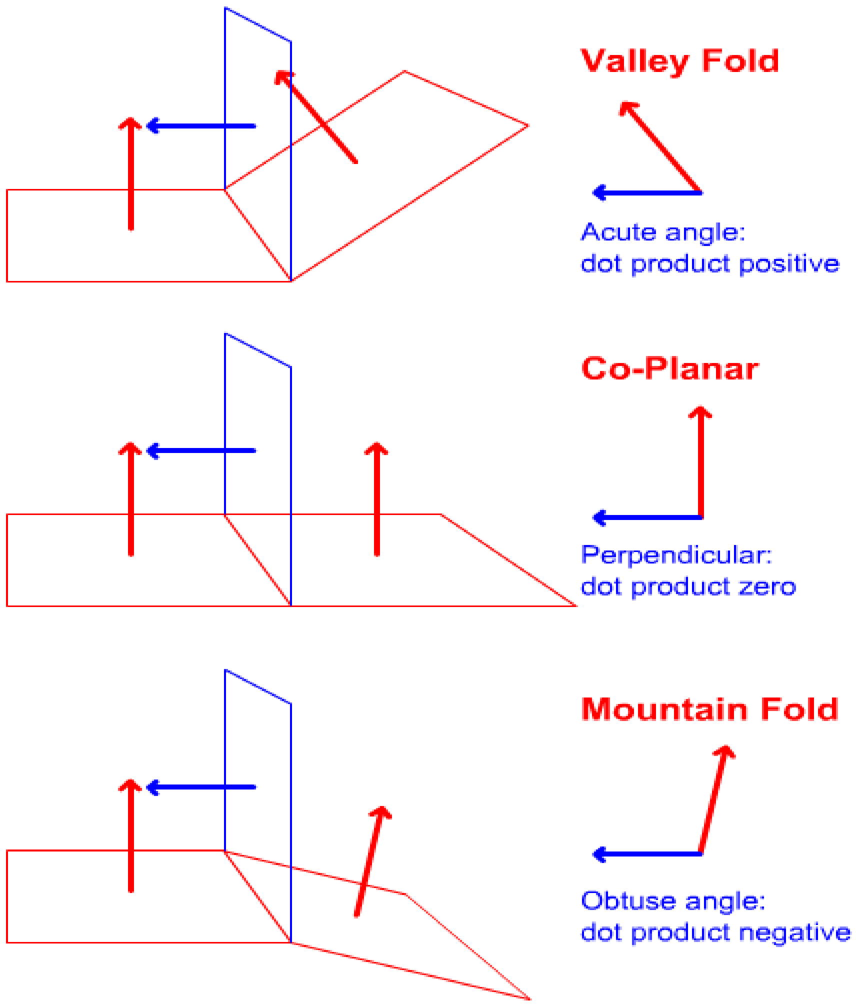
by substituting in (1) we get $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}$=3

5. Prove that the distance between parallel planes $2x - 2y + z + 3 = 0, 4x - 4y + 2z + 5 = 0$ is $1/6$.

Sol: the given planes are $2x - 2y + z + 3 = 0, 2x - 2y + z + 5/2 = 0$

the distance is $\frac{|3-5/2|}{\sqrt{4+4+1}} = \frac{1/2}{3} = 1/6$

Planes bisecting the angles between two planes



$\pi_1 = a_1x + b_1y + c_1z + d_1 = 0, \pi_2 = a_2x + b_2y + c_2z + d_2 = 0$ *and* $d_1d_2>0$.

Equation to the plane bisecting the angle containing the origin between the planes is

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}} = +\frac{a_2x+b_2y+c_2z+d_2}{a_2{}^2+b_2{}^2+c_2{}^2} \quad d_1>0, d_2>0, d_1<0, d_2<0$$

And to the plane bisecting the other angle between the planes is

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}} = -\frac{a_2x+b_2y+c_2z+d_2}{a_2{}^2+b_2{}^2+c_2{}^2} \quad d_1>0, d_2>0, d_1<0, d_2<0$$

- If $d_1>0, d_2<0$ or $d_1<0, d_2>0$

Equation to the plane bisecting the angle containing the origin between the planes is

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}} = -\frac{a_2x+b_2y+c_2z+d_2}{a_2{}^2+b_2{}^2+c_2{}^2}$$

And to the plane bisecting the other angle between the planes is

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}} = +\frac{a_2x+b_2y+c_2z+d_2}{a_2{}^2+b_2{}^2+c_2{}^2}$$